10th Workshop on Geometry and Dynamical Systems

Virtual Event, April 20-22, 2021

Presentation

The Workshop on Dynamical Systems and Geometry (WDSG) is intended to be an space for the discussion about problems and research results related with the applications of geometric, analytical and numerical methods in the study of dynamical systems, partial differential equations, and the equations of mathematical physics.

The topics usually covered on the WDSG are related with the following fields:

- Hamiltonian Systems and Perturbation Theory
- Holomorphic Dynamics
- Symplectic Geometry and Poisson Geometry
- Lie Algebroids and Lie Grupoids
- Dirac Structures
- Integrability and Supersymmetry
- Classical and Quantum Mechanics
- Supermanifolds
- Partial Differential Equations
- Computational Mechanics

The 10th edition of the WDSG will take place from April 20 to 22 through the zoom platform. The event is organized by the Department of Mathematics at University of Sonora (UNISON), Hermosillo, Mexico; the Mathematics Institute at the National Autonomous University of Mexico (UNAM); and the Center for Research in Mathematics (CIMAT), Guanajuato, Mexico.

Registration link:
`https://zoom.us/webinar/register/WN_BdSet2HBR32eUSbzJ5QE0w`

Organizing Committee

Misael Avendaño Camacho
Guillermo Dávila Rascón
Rubén Flores Espinoza
Yury Vorobev
José Crispín Ruíz Pantaleón
Carlos Villegas Blas

Scientific Committee

José Antonio Vallejo Rodríguez
Andrés Pedroza
Yury Vorobev

Scientific program

Schedule

The workshop will officially run on Hermosillo, Sonora Time (GMT-7)

Time (Pacific zone time, GMT-7)*	**Tue, April 20**	**Wed, April 21**	**Thu, April 22**
9:30 - 10:20	Javier de Lucas	Eva Miranda	Manuel de León
10:20 - 10:50	Eduardo Velasco	Andrés Pedroza	Jesús Muciño
10:50 - 11:00	Break		
11:00 - 11:50	Alan Weinstein	Luis García Naranjo	Xavier Gómez Mont
11:50 - 12:20	José Ruíz	Dennise García	Misael Avendaño
12:20 - 12:50			Ixchel Rodríguez

* For reference, some local times: 9:30 Los Angeles, 11:30 Mexico City, 12:30 New York, 13:30 Rio de Janeiro, 17:30 London, 18:30 Rome, 19:30 Moscow.

Titles and abstracts

A time-dependent energy-momentum method

Javier de Lucas
Department of Mathematical Methods in Physics, University of Warsaw

In this talk, I will devise a non-autonomous generalization of the so-called energy momentum-method so as to study the stability of Marsden–Weinstein reductions of a non-autonomous Hamiltonian system relative to a symplectic form with a Lie group of Hamiltonian symmetries. We characterize the so-called relative equilibrium points, that, after reduction, are equilibrium points of the dynamics of the reduced Hamiltonian system. Next, we provide conditions ensuring the stability around equilibrium points of the dynamics of the reduced Hamiltonian systems. As a byproduct, a geometrical extension of notions and results from non-autonomous stability theory on linear spaces to manifolds is provided. As applications, we study different non-autonomous Hamilton systems of physical and mathematical interest.

Hamiltonization criteria by rank-two Poisson stuctures

Eduardo Velasco Barreras

Universidad de Sonora

Given a vector field on a smooth manifold, we consider the problem of endowing it with a Hamiltonian structure by means of a Poisson structure. We begin by considering the case of vector fields on orientable manifolds, and present some hamiltonization criteria in terms of the existence of sufficient first integrals. We also consider the case of decomposable Poisson structures. By means of a generalization of the Hojman construction, we show that a vector field is hamiltonizable if it admits a first integral regular on its support and a Riemannian metric that is (transversally) invariant. This allows to provide some hamiltonization criteria for periodic vector fields. Finally, we give some remarks on how these ideas can be applied to the problem of hamiltonization of Lie group actions.

From symmetry to constraints in general relativity

Alan Weinstein

University of California, Berkeley; Stanford University

In general relativity, the gravitational field is a Lorentz metric on a 4-dimensional space-time manifold. The Einstein field equations may be expressed, at least locally in time, as the trajectories of a hamiltonian system on the cotangent bundle T^*R of the (infinite dimensional) manifold R of riemannian metrics on a 3-dimensional "time slice", with initial values constrained to a (singular) submanifold C of T^*R.

Geometric properties of C suggest that the constraints should be related to the symmetry group of the Einstein equations, consisting of the diffeomorphisms of space-time. But this group does not act on R, since it does not act on an individual time slice. Blohmann, Fernandes, and I showed that the algebraic structure of the constraints is in fact related to a group*oid* of diffeomorphisms between pairs of time slices, but a direct connection between the constraints and this groupoid was not found.

In work with Blohmann, we developed a notion of "hamiltonian Lie algebroid,ïntended to provide a framework to study the interaction between groupoids and symplectic structures. More recently, in ongoing work with Blohmann and Schiavina, we are attempting to find a hamiltonian Lie algebroid which explains the properties of the constraints. In particular, we have constructed an interesting Lie algebroid-like object which lives over the product of T^*R with a space whose coordinates are infinitesimal of first order; i.e., all products among them are zero. This space is a slight modification of a supermanifold which arises naturally via the BFV (Batalin-Fradkin-Vilkovisky) theory of boundary value problems for field theories with symmetries which do not preserve the boundary. We are still searching for the relevant hamiltonian structure.

On the Infinitesimal Geometry of Poisson Submanifolds

José Crispín Ruíz Pantaleón
Instituto de Matemáticas, UNAM

We present a geometric characterization of the Poisson algebras associated to the infinitesimal data of Poisson submanifolds by using the notion of contravariant derivative. One of our motivations is to provide a suitable framework for constructing linearized models of Poisson structures around Poisson submanifolds. In particular, we are interested in the study of such linearized models within the class of the so-called almost-coupling Poisson structures. An alternative approach to the construction of local Poisson models, based on the theory of symplectic Lie groupoids, was also suggested by R. Fernandes and I. Marcut.

Looking at Euler flows through a contact mirror: Universality and Turing completeness

Eva Miranda
Universidad Politécnica de Cataluña

The dynamics of an inviscid and incompressible fluid flow on a Riemannian manifold is governed by the Euler equations. Recently, Tao launched a programme to address the global existence problem for the Euler and Navier-Stokes equations based on the concept of universality. Inspired by this proposal, we show that the stationary Euler equations exhibit several universality features, in the sense that, any non-autonomous flow on a compact manifold can be extended to a smooth stationary solution of the Euler equations on some Riemannian manifold of possibly higher dimension. A key point in the proof is looking at the h-principle in contact geometry through a contact mirror, unveiled by Sullivan, Etnyre and Ghrist more than two decades ago.

Time permitting, we end up this talk addressing an apparently different question: What kind of physics might be non-computational? The universality result above yields the Turing completeness of the steady Euler flows on a 17-dimensional sphere. But, can this result be improved?

This talk is based on joint work with Robert Cardona, Daniel Peralta-Salas and Fran Presas (arXiv:1911.01963).

On the rank of the fundamental group of $\mathrm{Ham}(M,\omega)$

Andrés Pedroza
Universidad de Colima

The homotopy type of the group of Hamiltonian diffeomorphisms of a symplectic manifold is a far reaching problem in symplectic topology. Only for a few symplectic manifolds the homotopy type of $\mathrm{Ham}(M,\omega)$ is known.

Let k be a positive integer. We will construct a simply connected four-dimensional closed symplectic manifold such that the rank of the fundamental group of $\mathrm{Ham}(M,\omega)$ is at least k.

Platonic solids and symmetric solutions of the N-vortex problem on the sphere

Luis García Naranjo
University of Padua

We consider the N-vortex problem on the sphere assuming that all vortices have equal strength. We develop a theoretical framework to analyse solutions of the equations of motion with prescribed symmetries. Our construction relies on the discrete reduction of the system by twisted subgroups of the full symmetry group that rotates and permutes the vortices. Our approach formalises and extends ideas outlined previously by Tokieda (C. R. Acad. Sci., Paris I 333 (2001)) and Souliere and Tokieda (J. Fluid Mech. 460 (2002)) and allows us to prove the existence of several 1-parameter families of periodic orbits. These families either emanate from equilibria or converge to collisions possessing a specific symmetry. Our results are applied to show existence of families of small non-linear oscillations emanating from the platonic solid equilibria. This is a joint work with Carlos García-Azpeitia (IIMAS, UNAM).

Poisson structures on trivial extension algebras

Cynthia Dennise García Beltrán
Cátedras CONACYT-Universidad de Sonora

We describe a class of Poisson structures on trivial extension algebras which generalize some known structures induced by Poisson modules. First, we show that there exists a one-to-one correspondence between this class of Poisson structures and some data involving (not necessarily flat) contravariant derivatives and then give a formulation of this result in terms of Lie algebroids. As an application, an abstract algebraic setting of the Hamiltonization problem for a special family of derivations of a Poisson algebra is discussed. Finally, some examples arising from Poisson modules and Poisson submanifolds are given.

The Hamilton-Jacobi equation for contact Hamiltonian systems

Manuel de León
ICMAT-Consejo Superior de Investigaciones Científicas and Real Academia de Ciencias

In this talk we will first give an introduction to contact Hamiltonian systems, and then we will discuss the Hamilton-Jacobi equation for this type of systems. More specifically, we will consider two types of dynamics, that determined by the Hamiltonian vector field and that of the so-called evolution vector field (the latter relevant in applications to thermodynamics). We will conclude with some examples.

On the classification of polynomials and Hamiltonian vector fields on the plane

Jesús Muciño
Centro de Ciencias Matemáticas UNAM, Morelia

We study the real or complex polynomials of two variables that are uniquely determined by the position of their critical points. A classification for low degree is provided (using affine transformations of the plane).

On the Geometry of a Bilinear Form Associated to a Plane Curve Singularity over $\mathbb{C}$

Xavier Gómez-Mont
Centro de Investigación en Matemáticas

A polynomial in 2 complex variables $F(z_1, z_2)$ defines via its level curves $C_t := F^{-1}(t)$ a family of Riemann Surfaces that have a singularity at the critical points. For non-critical values $t \in \mathbb{C}$ we have a locally trivial fibration for $\{C_t\}$. When we tend from regular values to a critucal value t_0, part of the topology of C_t is going to contract to the singular point (supposing it is isolated). We measure the topology with the 1-homology group $H_1(C_t, \mathbb{Z})$ of C_t, whose dimension is the first Betti number of C_t. The vanishing homology consists of those 1-cycles in C_t which are being contracted to the singular point of C_{t_0}.

The (algebraic) monodromy map

$$T : H_1(C_t, \mathbb{Z}) \longrightarrow H_1(C_t, \mathbb{Z})$$

is the automorphism of $H_1(C_t, \mathbb{Z})$ obtained from the fibration given by F by going around a small circle centered at t_0. The invariant 1-cycles form the eigenspace associated to the eigenvalue 1 of T, and the .ºrthogonal.ºf the invariant cycles, where the orthgonality in 1-homology comes from the bilinear form obtained by intersection of 1-cycles, is the space generated by the vanishing cycles.

The monodromy transformation T decomposes additively as a diagonalizable (semi-simple) part and a nilpotent part N. The bilinear form that we consider is the one which is obtained from intersection product of $H_1(C_t, \mathbb{Z})$ by applying N previously to one of the factors:

$$< N*, * >: H_1(C_t, \mathbb{Z}) \times H_1(C_t, \mathbb{Z}) \longrightarrow \mathbb{Q},$$

to obtain a symmetric bilinear form which degenerates on $Ker(N)$ and hence defines a symetric non-degenerate bilinear form

$$\frac{H_1(C_t, \mathbb{Z})}{Ker(N)} \times \frac{H_1(C_t, \mathbb{Z})}{Ker(N)} \longrightarrow \mathbb{Q}.$$

The aim of the presentation is to give a geometric interpretation of this symmetric non-degenerate bilinear form and prove, on using the geometric interpretation, that the it is positive definite.

The Theory of Plane Algebraic Curves provides a geometric monodromy map

$$\phi : C_t \longrightarrow C_t,$$

which induces the algebraic monodromy in 1-homology, which is such that ϕ^d is the identity except in a tubular neighbourhood of disjoint 1-cycles $\{\gamma_j\}$, whose union is ϕ-invariant, the vanishing cycles, where each ϕ^{d_j} is a Dehn twist to the right by a positive integer on a tubular neighbourhood. We give a geometric interpretation of $\frac{H_1(C_t, \mathbb{Z})}{Ker(N)}$, of N and on the bilinear form in terms of the geometry of ϕ.

This is a joint collaboration with L. Alanís, E. Artal, Ch. Bonatti, M. González-Villa y P. Portillo, and the details may be found in the Arxive server.

Some problems in semiclassical cuantization on slow-fast phase spaces

Misael Avendaño Camacho
Departamento de Matemáticas, Universidad de Sonora

Our point is to construct quasi-modes for semiclassical Weyl operators associated with a class of (non-integrable) Hamiltonian systems of adiabatic type on slow-fast phase spaces.

First, we show that there exists a family of Lagragian almost invariant 2-tori for a slow-fast Hamiltonian system with two degrees of freedom. Then, by using the Karasev integral ansatz we show that this family of Lagrangian 2-tori can be quantized in the semiclassical approximation. In particular, we describe adiabatic corrections to the Bohr-Sommerfeld quatization rule associated to the Hannay-Berry connection.

Classification results on four-dimensional homogeneous conformally Einstein manifolds and C-spaces

Ixchel Dzohara Gutiérrez Rodríguez
Universidad de Colima

The existence of Einstein representatives of a given conformal class of a metric gives rise to conformal generalizations of the Einstein condition [1]. The purpose in this talk is to review some classification results for some kinds of generalizations of Einstein metrics in the homogeneous setting. We will recall some known results and give explicit realizations of homogeneous conformally Einstein metrics and conformal C-spaces in dimension four [2].

References

[1] R. Gover and P.-A. Nagy, *Four-dimensional conformal C-spaces*, Quart. J. Math. 58 (2007), 443–462

[2] E. Calviño-Louzao, E. García-Río, I. Gutiérrez-Rodríguez, and R. Vázquez-Lorenzo, *Four-dimensional homogeneous conformal C-spaces*, Osaka J. Math., to appear.
